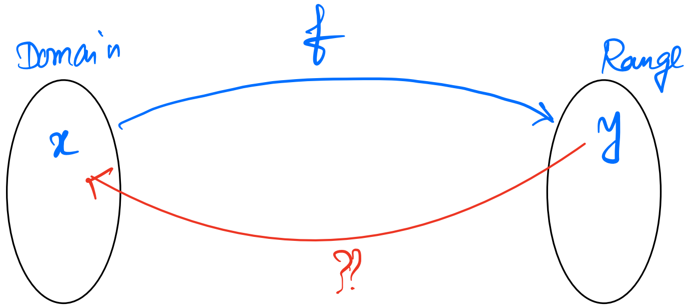
Inverse Functions



Given $f: D \longrightarrow R$. If inverse function exists, then
D ↑ domain, R ↑ range

it is denoted by $f^{-1}: R \longrightarrow D$ s.t

$$f^{-1}(f(x)) = x \quad , \text{ for all } x \text{ in } D$$

$$\& \quad f(f^{-1}(y)) = y \quad , \text{ for all } y \text{ in } R.$$

Note: $f: D \rightarrow R$ a constant function cannot have inverse function.

One to One function

f is one to one on D or simply f is 1-1 if

$$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$$

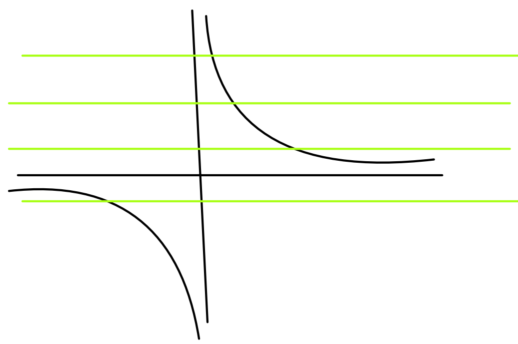
or

$$x_1 = x_2 \Rightarrow f(x_1) = f(x_2)$$

Eg. $f(x) = x^2$ is one-to-one on $[0, \infty)$, but not in $(-\infty, \infty)$.

Horizontal Line Test

f is one-to-one $\Leftrightarrow$ every horizontal line intersects the graph of f atmost one point.



How to find inverse function?

① Solve the equation $y = f(x)$ for x.

② Interchange x & y & write $y = f^{-1}(x)$.

Example: $f(x) = 3x - 4$, $f^{-1}(x) = \frac{1}{3}x + \frac{4}{3}$

Inverse Trigonometric Functions

$\sin^{-1}$: $D = \{x \mid -1 \leq x \leq 1\} \longrightarrow \{y \mid -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}\}$

$\sin^{-1}(x) = y \Leftrightarrow \sin(y) = x$, $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$.

$\cos^{-1}$: $D = \{x \mid -1 \leq x \leq 1\} \longrightarrow \{y \mid 0 \leq y \leq \pi\}$

$\cos^{-1}(x) = y \Leftrightarrow \cos(y) = x$, $0 \leq y \leq \pi$.

$\tan^{-1}$: $D = \{x \mid -\infty < x < \infty\} \longrightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\tan^{-1}(x) = y \Leftrightarrow \tan y = x \;,\; -\frac{\pi}{2} < y < \frac{\pi}{2}$

$\cot^{-1}$: $D = \{x \mid -\infty < x < \infty\} \longrightarrow (0, \pi)$

$\cot^{-1}(x) = y \Leftrightarrow \cot y = x \;,\; 0 < y < \pi$

$\csc^{-1}$: $D = \{x \mid |x| \geq 1\} \longrightarrow \left\{y \mid -\frac{\pi}{2} \leq y \leq \frac{\pi}{2} \;\&\; y \neq 0\right\}$

$\csc^{-1}(x) = y \Leftrightarrow \csc(y) = x \;,\; -\frac{\pi}{2} \leq y \leq \frac{\pi}{2} \;\&\; y \neq 0$

$\sec^{-1}$: $D = \{x \mid |x| \geq 1\} \longrightarrow \left\{y \mid 0 \leq y \leq \pi \;\&\; y \neq \frac{\pi}{2}\right\}$

$\sec^{-1}(x) = y \Leftrightarrow \sec(y) = x \;,\; 0 \leq y \leq \pi \;\&\; y \neq \frac{\pi}{2}$